Blind Algorithm Development for Peak to Average Power Ratio Reduction in OFDM Systems under Frequency Selective Channels

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ABSTRACT

One major drawback of orthogonal frequency division multiplexing (OFDM) system is peak to average power ratio (PAPR). This effect causes high power amplifier (HPA) to introduce intermodulation and out of band radiation as the signal goes through, thus degrades the performance of OFDM systems. This paper proposes blind algorithms which takes advantage of signal transformation technique and signal distortion technique. Simulation results show that at complementary cumulative distribution function (CCDF) level of $10^{-3}$, the proposed algorithm achieved 3.2 dB PAPR improvement compared to discrete Fourier transform with interleaved frequency division multiple access (DFT-IFDMA) based algorithm. The bit error rate (BER) performance has degraded by 2 dB compared to the original OFDM signal with no distortion under frequency selective channel (FCS) at BER of $10^{-4}$. These presented results, mark this algorithm as a better candidate for PAPR reduction algorithm in long term evolution (LTE) network. Under AWGN channels, the proposed algorithm performs better both in low and high signal power values. Under frequency selective channels, the existing and proposed algorithm converges after 10 dB of signal to noise power values. The low BER transmissions at low signal power values signify energy efficiency, ideal for portable wireless devices with limited battery power.

Keywords: Bit Error Rate, Frequency Selective Channel, High power amplifier, Orthogonal frequency division multiplexing (OFDM), Peak to average power ratio (PAPR).

INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a transmission scheme which uses multi-carriers to overcome severe environmental challenges which affect wireless communications spectral efficiency performance (Singh and Sharma, 2015). Its multicarrier nature has managed to attract several technologies, both wired and wireless communication systems. These includes, Digital Audio and Video Broadcasting, Digital subscriber line using digital multitone, Wireless Local Area Network (WLAN) such as IEEE 802.11, High Performance Radio Local Area Network (HIPERLAN) and Multimedia Mobile Access Communication (MMAC), wireless broadband service, WiMAX IEEE 802.16 and other new generation cellular network such as LTE (Singh and Sharma, 2015; Kondamuri and Sundru, 2018; Liu et al., 2018). However, this technology has several drawbacks including high sensitivity to frequency offset, spectral null in the channel and high peak to average power ratio (PAPR) (Yang, 2005). Since several transmission schemes have adopted OFDM, it is of interest to address the mentioned drawbacks.
A transmitted OFDM signal is a superposition of multiple orthogonal subcarriers which at a given time can have the same phase thus, add up and generate peaks whose power is large when compared to average power of the signal. Signal with large PAPR require power amplifier with large range of input backoff (IBO) for linear amplification. Linear high-power amplifier (L-HPA) is very expensive and they are inefficient (Salah et al., 2009). It is of interest to use non-linear HPA which has small range of IBO for wide spread of technology. Therefore, it is necessary to reduce PAPR by appropriate methods while achieving close to optimal system performance (Wetz et al., 2006; Litsyn, 2007). PAPR causes non-linear HPA to radiate in-band and out-of-band radiation when the transmitted signal passes through it (Hao et al., 2019). The out-of-band (OOB) effect is of concern to a regulatory authority such as Tanzania Communication Regulatory Authority (TCRA) for the case of Tanzania since the licence of the broadcast is breached. Also, at the receiver end, the detection efficiency is highly sensitive to devices such as analog to digital converters (ADC) which is highly affected by larger PAPR.

There are two methods to mitigate this problem, signal processing technique and linear HPA (Chakrapani and Palanisamy, 2012; Myung et al., 2006). The latter is not advisable as it is very sensitive and it has poor efficiency as stated earlier. It is better to choose signal processing techniques. Signal processing techniques are in three categories. These are transmitter-based technique, signal transformation technique and receiver-based technique. Transmitter based technique has two categories; distortion technique and distortion-less techniques (Welden and Steendam, 2008; Chakrapani and Palanisamy, 2012). In receiver-based technique, there are maximum likelihood and signal reconstruction (Hei et al., 2017). Signal transformation technique involves transforming the signal before HPA in transmission, and the inverse is performed before demodulation.

Godwin M. Gadiel, Kwame S. Ibwe and M. M. Kissaka

In literature, researches have presented a lot of signal processing techniques. These includes, clipping (Mounir et al., 2017; Ibwe, 2019), peak windowing, non-linear companding technique (NTC), partial transmit sequence (PTS), selected mapping (SLM), selective mapping, tone reservation (TR), tone injection (TI), clipping and filtering (CF), discrete fourier transform spreading, discrete cosine transform, peak windowing (Cha et al., 2008; van Welden and Steendam, 2008; Chen et al., 2009; Sharma and Verma, 2011; Vittal, 2012; Chakrapani and Palanisamy, 2012; Wang et al., 2016). But still, a 0 dB PAPR system is far within reach. Most of the proposed algorithms use side information, which consumes significant part of the bandwidth to transfer dummy information. A 0 dB PAPR signifies that, HPA can operate at an optimal point, maximizing the average transmit power and maximizing HPA efficiency (Chakrapani et al., 2012). This paper proposes a blind algorithm in which, by applying signal transformation before signal distortion, it will cause less distortion to the resulting signal. The PAPR reduction is characterized by considering the cumulative distribution function (CCDF) metric in accordance with the general way of presenting results on the subject (Ann and Jose, 2016). The proposed algorithm has shown improved PAPR with less distortion in BER performance. The error rate performance is assessed in terms of bit mean squared error to noise power ratio. These results are obtained using simulation platform, MATLAB®. To test for the performance of the resultant transmit signal AWGN and frequency selective wireless channel environments are used. The channel models used are from Proakis (2001).

METHODS AND MATERIALS

OFDM Signal

In OFDM a group of binary data is first modulated by a modulation scheme chosen for
a specific environment. A fast serial data is converted to low data rate parallel bits of length \( N \) where \( N \) is the number of samples of a given data stream. The \( N \) chunks of parallel bits are expressed as shown in equation (1) and (2)

\[
x(n) = X(k)e^{j2\pi kn/N} \quad k = 0, 1, ..., N-1
\]

whereby each symbol modulating one of subcarriers \( f_k = k\Delta f \) where \( \Delta f = 1/NT_2 \) and \( T_2 \) is the original symbol period. Mapping of data into subcarriers is performed by a unitary transformation function IDFT and the resulting time domain signal is given in equation (2).

\[
x(n) = \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N} \quad n = 0, 1, 2, ..., N-1
\]

**PAPR Definition**

Continuous time PAPR of OFDM signal \( x(t) \) is defined as the ratio between the maximum instantaneous power and its average power. In practice, it is more concerned in reducing PAPR of a continuous signal since the cost and power dissipation of analog components always dominates. But most systems are in discrete time signal model. Therefore, it is required to sample continuous time signal \( x(t) \) with an overlapping factor \( L \). The resulting equation is given in equation (3).

\[
x(n) = \sum_{k=0}^{\frac{N-1}{L}} X(k)e^{j2\pi kn/LN} \quad 0 \leq n \leq LN-1
\]

In the literature (Sharif *et al.*, 2002) \( L \geq 4 \) is sufficient enough to accurately estimate the PAPR of a continuous signal from the digital signal. The PAPR reduction performance is evaluated by complementary cumulative distribution function (CCDF), which is defined as the probability that the PAPR of \( x \) exceeds a given clip level \( y \) i.e \( \text{CCDF}_{\text{PAPR}(x)} = \Pr(\text{PAPR}(x) > y) \).

**DFT Spreading**

DFT spreading is an algorithm that performs pre-conditioning/spreading of data at the transmitter and resulting in diminished value of PAPR system as the pre-coding is DFT based. The notion of introducing DFT before IDFT creates an identity matrix which is a property of a single carrier and thus reduced PAPR. An M point DFT spread is performed to \( X(n) \) signal by using a block matrix defined by equation (4).

\[
A_{\text{DFT}} = \begin{bmatrix}
\cos\frac{2\pi km}{M} \\
\end{bmatrix}
\quad k, m = 0, 1, 2, ..., M-1
\]

The resulting vector signal from DFT spreading is given by equation (5).

\[
X'(k) = \sum_{m=0}^{\frac{M-1}{L}} X(m)e^{j2\pi km/M}
\]

The resulting PAPR value highly depends on the type of subcarrier mapping scheme. There are three types of subcarrier mapping. These are interleaved frequency division multiple access (IFDMA), localized frequency division multiple access (LFDMA), and distributed frequency division multiple access (DFDMA) (Sohl and Klein, 2007). Figure 1 describe the three subcarrier mapping schemes.

The three-subcarrier mapping, IFDMA, DFDMA and LFDMA are mathematically represented by equation (6) through (8) respectively.

\[
\tilde{X}(k) = \begin{cases}
\frac{X_{k-L}}{ML} & \text{if } s = ML + 1 \\
0 & \text{otherwise}
\end{cases}
\]

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\frac{X_{k-L}}{ML} & \text{if } s = ML + 1 \\
0 & \text{otherwise}
\end{cases}
\]

\[
\tilde{X}(k) = \begin{cases}
\frac{X_{k-L}}{ML} & \text{if } s = ML + 1 \\
0 & \text{otherwise}
\end{cases}
\]

The resulting time domain signal after IFFT process for IFDMA, DFDMA and LFDMA are given by equation 9 through 11, respectively.

\[
X(m) = \frac{1}{L}e^{j2\pi f_m/\Delta f}
\]

\[
\tilde{X}(k) = \begin{cases}
\frac{X_{k-L}}{ML} & \text{if } s = ML + 1 \\
0 & \text{otherwise}
\end{cases}
\]

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0 & \text{otherwise}
\end{cases}
\]

\[
X(m) = \frac{1}{L}e^{j2\pi f_m/\Delta f}
\]
Figure 1: IFDMA, DFDMA and LFDMA DFT spreading mapping (Gouda et al., 2013)

It can be realized from equation 9 through 11 that, the resulting time domain signal after IFFT process is the scaled version of the original signal (Gouda et al., 2013).

**Peak Windowing**

Peak windowing is applied to the output of the IFFT process to further reduce the PAPR to a significant value. A signal is compared with a threshold, then a window which smooths the peaks according to the limit is applied. The width of the window plays a significant role in BER performance, thus checking on the size of the window is a significant concern in the system. The peak windowing algorithm is described in Table 1 (Chen et al., 2009).

Figure 2 gives a brief description of Peak Windowing algorithm. The smoothing function \( s(m) \) can be realized as a FIR filter generated from a convolution of the produced impulse response and the selected window function. The function \( s(m) \) is given by equation (12) (Cha et al., 2008).

\[
s(m) = 1 - \sum_{k=-\infty}^{\infty} \{1-c(m)\}w(m-k)
\]  

(12)

Figure 3 shows the simulation results of equation (11), when a Kaiser window of length 8, together with a clipping threshold of \( \lambda = 0.7 \) and shape parameter \( \beta = 3.5 \).
Proposed Algorithm

This paper proposes an algorithm that combines two techniques, DFT spreading and peak windowing whose block diagram is presented in Figure 4. The resulting time domain signal of a DFT based OFDM is a scaled version of the original signal. Then if a distortion technique is applied to improve PAPR further, the performance degradation is less compared to if only distortion technique is used to the system. With this assumption in mind, it is expected the proposed algorithm will have improved performance. The proposed algorithm is blind, which ensures high spectral efficiency since no side information is required at the receiver for decoding purposes. Table 2 shows the signal flow model of the proposed algorithm.
Table 2: Proposed algorithm

Step 1: The sequence $x(n)$ from the modulator is transformed using a DFT matrix followed by zero insertion depending on IFDMA, DFDMA and LFDMA.

Step 2: An IFFT is performed and resulting in equations 8, 9, or 10 for interleaved, distributed and localized mapping respectively.

Step 3: After the addition of a cyclic prefix, the peak windowing algorithm is then applied to the resulting time domain signal to further minimize high peaks further.

Step 4: At the receiver, the cyclic prefix is removed, and the FFT transform is applied to the received signal.

Step 5: Padded zeros removed, followed by inverse DFT spreading. Then the signal is de-mapped to the bit stream.

RESULTS AND DISCUSSION

In this section, the proposed algorithm is simulated in OFDM system under frequency selective fading channel. The channel is modelled as a FIR filter with sufficient taps to accommodate different path delays for the worst-case transmission scenarios. The system specification is provided in Table 3. The MATLAB® software is used to simulate the set experiments and results of the performance of the proposed algorithm presented in plot forms.

Table 3: System Specification

<table>
<thead>
<tr>
<th>Parameter(s)</th>
<th>Values(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mapping</td>
<td>QPSK</td>
</tr>
<tr>
<td>IFFT Point</td>
<td>512</td>
</tr>
<tr>
<td>Oversampling</td>
<td>4</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Carrier Frequency</td>
<td>3 GHz</td>
</tr>
<tr>
<td>Cyclic Prefix</td>
<td>25%</td>
</tr>
</tbody>
</table>

First, the performance gain of the two independent algorithms which have been used in the proposed system is established. Figure 5 shows a comparison of CCDF between the OFDM and DFT spreading OFDM system. Original OFDM system
has a PAPR value of 9.9 dB at CCDF of $10^{-3}$, while that of DFT spreading has the values 7.6 dB, 7.1 dB and 6.5 dB for DFDMA, LFDMA and IFDMA, respectively. The maximum improvement of 3.4 dB form IFDMA and a minimum value of 2.3 dB from DFDMA is observed.

Figure 6 shows PAPR reduction performance of peak windowing algorithm for different windowing, for the system specification in Table 3. A constant value of window-length, set to 100, was used for Hamming, Hanning and Kaiser windows. In the case of the Kaiser window, spectral shaper ($\beta = 3.5$) is used. There is an improvement of PAPR at CCDF = $10^{-3}$ as follows; for Hamming window, there is an improvement of 1.2 dB, for Hanning window there is an improvement of 1.1 dB, and for Kaiser window, there is an improvement of 1.3 dB compared to an original OFDM system.

![Figure 5: PAPR reduction performance of DFT-spreading algorithms for the system specifications in Table 3](image)

**Figure 5: PAPR reduction performance of DFT-spreading algorithms for the system specifications in Table 3**
Figure 7 shows the CCDF performance of the proposed algorithm for the system specified in Table 1. At CCDF = 10^-3, LFDMA-DFT-spreading and peak windowing have improved PAPR to 6.4 dB, DFDMA-DFT-spreading and peak windowing have improved PAPR to 6 dB, and the best performance is observed by IFDMA-DFT-spreading and peak windowing which achieved a PAPR of 3.35 dB. The proposed algorithms have by far improve the value of PAPR compared to DFT-spreading and peak windowing algorithms. In this simulation, Kaiser Window, with window-length of 100 and spectral shaper of β = 3.5 have been used since it has better performance in an extensive range of CCDF compared to other windows as can be seen in Figure 6.

The BER performance of the proposed algorithm is compared with the conventional algorithms. Figure 8 shows, BER performance for a system described in Table 1, with different PAPR reduction algorithms. The proposed algorithm, IFDMA/PW-k, has higher performance compared to PW algorithm, and less performance compared to DFT-OFDM algorithm. The proposed algorithm has less performance compared to the latter, due to the distortion effect introduced by PW in the proposed algorithm. The performance of proposed IFDMA-PW algorithm is compared with the work done by Mounir et al. (2017) under AWGN and frequency selective channel conditions. Figure 9 shows the better performance of proposed algorithm for both small and larger values of signal power. In Figure 10, the performance of proposed algorithm and that proposed in Mounir et al. (2017) both converge after 10 dB of signal to noise power values. This results from the fact that Mounir et al. (2017) used a deliberate clipping with rectangular windowing method whose effects are similarly countered as the proposed algorithm at higher signal power values.
Figure 7: PAPR reduction performance of the proposed algorithm for the system specified in Table 3

Figure 8: BER performance versus signal to noise ratio for different algorithms
Figure 9: Performance of the proposed algorithm over AWGN channel

Figure 10: Performance of the proposed algorithm over Frequency Selective channel
CONCLUSIONS

This paper has presented a blind algorithm which employs DFT spreading and peak windowing for PAPR reduction. The proposed algorithm achieves low PAPR in low signal power values compared to DFT spreading as well as peak windowing algorithm. In particular, at a CCDF of $10^{-4}$, the proposed algorithm achieves a PAPR of 3.5 dB, while DFT spreading and PW-k achieves 6.8 dB and 9.3 dB, respectively. Furthermore, the proposed algorithm has low BER performance compared to the PW algorithm. Specifically, at an SNR of 14 dB, the proposed algorithm IFDMA/PW-k achieves a BER of $10^{-5}$ while PW achieves BER of $10^{-4}$. Low BER signifies the suitability of the algorithm to be used in harsh environments like frequency selective channels without degrading the battery efficiency of portable wireless devices. The DFT spreading is gaining popularity in LTE systems as one of the suitable radio technologies for uplink transmissions. Therefore, the proposed algorithm will be a useful alternative for energy efficient transmissions. Furthermore, the performance of the proposed algorithm, IFDMA-PW outperformed the existing one presented in Mounir et al. (2017) under AWGN channel. In frequency selective channels, the performance of both algorithms converged after 10 dB of signal to noise power ratio values. In future, optimization of the performance of the proposed algorithm for highly time and frequency changing channels could be researched. This will shed more light towards the characterization of both doubly dispersive channels and multicarrier transmission techniques.

REFERENCES


